The Ultimate in Anty-Particles

Ian Stewart

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Mathematics Institute
University of Warwick
Coventry CV4 7AL
UK

The hotel lobby was in a state of pandemonium, with suitcases and rucksacks piled all over the place. The line at reception trailed back to the revolving doors — and through them, which is a tricky thing for a line to do. I wondered for the hundredth time whether I'd been in my right mind even to consider attending the Quinquennial SPAM World Convention, but it was too late now to change my mind because I'd agreed to give a seminar.

Fortunately I prefer to travel light — a credit card, a change of underwear, and a pocketful of coins for the Laundromat. I decided to skip the check-in until everybody else was occupied with lining up for dinner, and headed for the hotel bar.

About half the participants had had the same idea. I sharpened my elbows and got stuck in.

The Society for Philosophizing about Mathematics isn't usually quite that disorganized. But its most ambitious meeting normally consists of half a dozen people getting together once a month in somebody's house and most of us can organize that kind of event. It's not easy to get funding for mathematical philosophizing, so what had once been an annual binge had slipped to once every five years. You don't get much continuity with a five year gap between conventions, so the committees tend to repeat the same mistakes.

Such as telling everybody to arrive at the same time.

A miraculous empty seat appeared and I sat down to fasten it in place before it got away. Immediately to my left, a smartly dressed woman in her mid-sixties was engaged in a heated discussion with what appeared to be the epitome of teenage grunge. He was extolling the virtues of Mahabhava's symplectic generalization of Gödel's Theorem, and she was trying to see how high she could pile all the glasses on the table. A thin woman with spiky hair wore a tee-shirt labelled 'watch this space'. There was an over-excited fuzzicist, who was busily explaining a flexible extension of conventional logic to an audience of skeptical constructivists who wanted to make it more rigid. A group of non-standard analysts were discussing whether a pizza of infinite radius could in principle be assembled from infinitely many infinitesimal choices of topping. And a nerdish type in one corner was madly tapping the keys of a laptop.

I grinned, and started to relax. The trip *was* going to be as intellectually rewarding as I'd hoped, after all. I introduced myself to spiky hair and tee-shirt, whose name turned out to be Louise. "I'm watching," I said.

"Uh?"

"This space."

"Oh, yeah."

"I'm not seeing anything unexpected."

She gave me a funny look as if she was trying to work out whether I was being sexist, and then laughed. "It's for The Equation," she said. "When they find it."

"The Equation?"

"Mind you, I'm not very hopeful of a quick breakthrough now that those idiots in Congress have cancelled the Superconducting Supercollider."

"Oh, *that* equation," I said. "The Theory of Everything." She was a fundamentalist; I should have twigged long before.

"You may scoff," she said. I started to shake my head, to indicate that scoffing had not been on my mind. "I merely believe that everything in the universe is governed by one fundamental law, and that the central aim of science must be to find out what it is."

"And then break it," suggested smartly dressed mid-sixties, who admitted to the name Juliet.

"Yes, but even if it exists, what makes you think it's a *mathematical* law?" teenage grunge queried.

"The entire course of scientific history," said Louise "has been the discovery of deeper and deeper mathematical patterns in nature. The very word 'laws' indicates a precision that can only be found in mathematics. Indeed we can define mathematics as the study of the logical consequences of simple, precise laws."

"What do you mean by 'logical'?" said one of the constructivists.

"What do you mean by 'consequences'?" said the second.

"What do you mean by — uh — 'mean'?" said the third. They were triplets — Tom, Dick, and Granville.

"What do you mean by 'precise'?" asked the fuzzicist, who was called Inez.

The convention was under way.

"The point is," said Louise, "that once we find the really basic underlying laws of nature, we can deduce everything else. Instead of a messy patchwork of approximate theories, we'll know the *truth*."

"I'm not sure there is such a thing as truth," said Inez. "How much truth? Half truth? Three quarter truth?"

"One minus epsilon truth where epsilon is infinitesimal," suggested one of the non-standard analysts.

"I think you're all on the wrong level of discussion," I said. "What bothers me is that phrase 'in principle'. It's only a hundred years or so since mathematicians proved that *in principle* the entire future of the universe is determined by its present state, and that led to a picture of a clockwork universe and the idea that simple laws necessarily generate simple behaviour. But when people started to think seriously about what's possible *in practice* they discovered chaos — simple laws can generate extremely complex behaviour, and deterministic systems can behave randomly.

"So, for the sake of argument, suppose you're right. Suppose that at base the universe really does obey a simple set of fundamental laws, and we find them. My question is, will that really help us understand the world in which we live?"

"Well, of course it will. First, it will provide a basic philosophical underpinning. Second, the entire behaviour of our human-sized world is necessarily implicit in the fundamental laws, so the laws will explain *everything*."

"In principle, maybe. But not in practice. For instance, in the human-scale universe cats like to chase mice. I wouldn't agree that your 'fundamental' laws explained that unless you could show me a convincing deduction, starting from your equations, and ending with the fact that cats like to chase mice. How do you plan to do that?"

"I suppose what you might do," Granville chipped in, "is put the basic equations on a computer, feed in the quantum-mechanical wave-functions of a cat and a mouse or whatever they are, and see what the computer predicts."

"Except," objected Tom, "that according to Schrödinger the wave-function can't even tell you whether the cat is alive or dead, let alone a mouse-fancier." He waved his glass excitedly and spilled his drink over Inez's head.

"I don't believe that kind of thinking makes the slightest sense," I said. "Even if you could carry the computations out, they would be impossibly huge and totally incomprehensible. No, the problem with Theories of Everything is that they start with the wrong concept of 'explanation'. An explanation is an *explicit* argument that leads from hypothesis to conclusion, not just a vague statement that the conclusion is implicit in the assumptions. And certainly not a stack of computer printout a thousand miles high that purports to render the implicit explicit."

"You mean," said Inez, "that since tomorrow's weather is implicit in today's, why do we bother with weather forecasts?" I nodded.

The nerd in the corner woke up. "Let me show you all something," he said. He leaned forward, cleared away several dozen empty glasses and the remains of an unidentifiable item of fast food, and plonked his laptop on the table. "Look at the screen." He thumbed the built-in trackball, and double-clicked. A fine grid of squares appeared. "That's an ant, OK?"

"Where?"

"In the middle, only it's invisible. But now I'll show it to you. He clicked again. *Something* rushed madly to and fro across the grid, leaving behind it a random trail of black and white squares. It continued for a minute or so, and then built a curiously patterned diagonal stripe and disappeared off the edge of the screen.

"Fascinating," said Louise. "Now, Ian, as I was saying about The Equation —"

"Louise," said the nerd, "if you'll let me explain what you just saw, you'll see that it is highly relevant to the topic under discussion."

"You always say that, Nathan, even when it's a game of Dougal the Dugong."

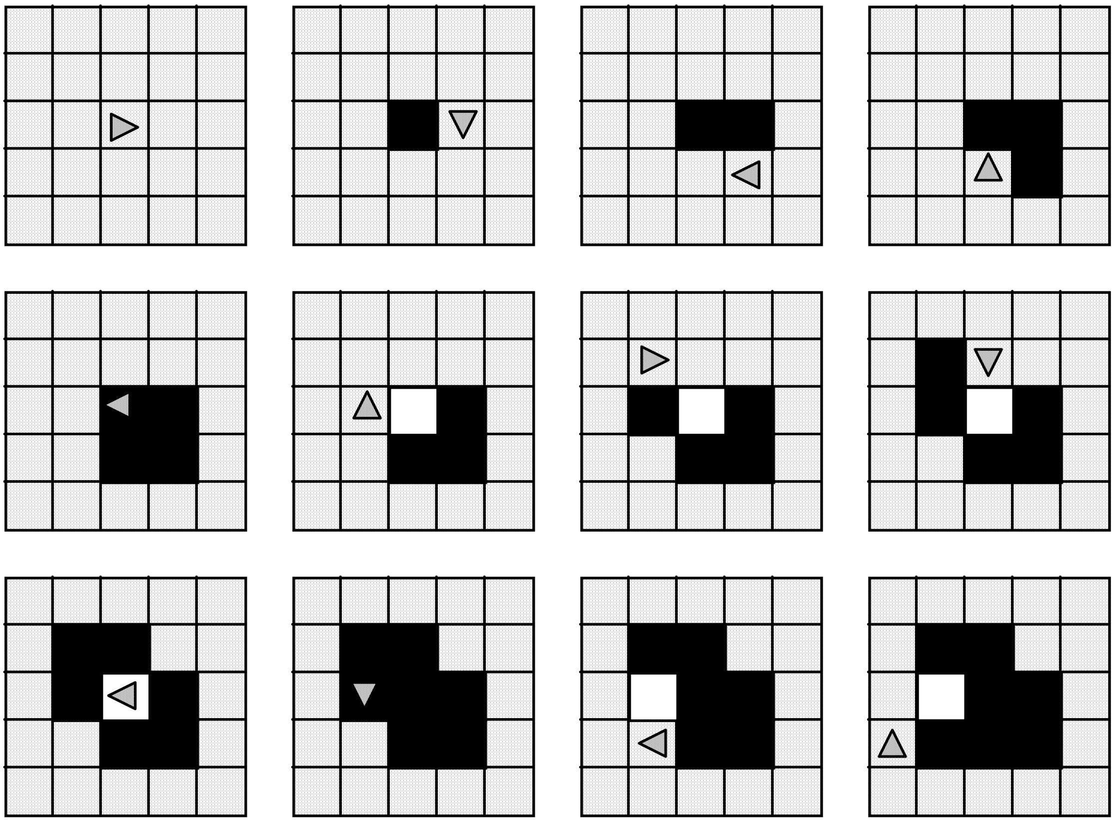
"Suspend judgment for a few minutes, and I'll justify my claim. What I've just shown you is a mathematical system known as Langton's Ant. It's an amazingly simple cellular automaton invented by Chris Langton of the Santa Fe Institute."

"The complexity mob?"

"Precisely. They look for large-scale regularities in complex systems. Langton's Ant is a simple example." The constructivists went off into a huddle about whether you could have a simple example of something complex, and everybody else ignored them. "The ant starts out on the central square, heading in some selected direction — say east. It moves one square in that direction, and looks at the color of the square it lands on — black or white. If it lands on a black square it paints it white and turns 90_ to the left. If it lands on a white square it paints it black and turns 90_ to the right. It keeps on following those same simple rules forever.

"You saw what happened when I started it on an all-white grid."

(See **Fig.1.**)



The first twelve steps followed by Langton's Ant (grey arrow). (For clarity, squares not yet visited are shown light grey: these should be treated as 'white' when applying the rules.)

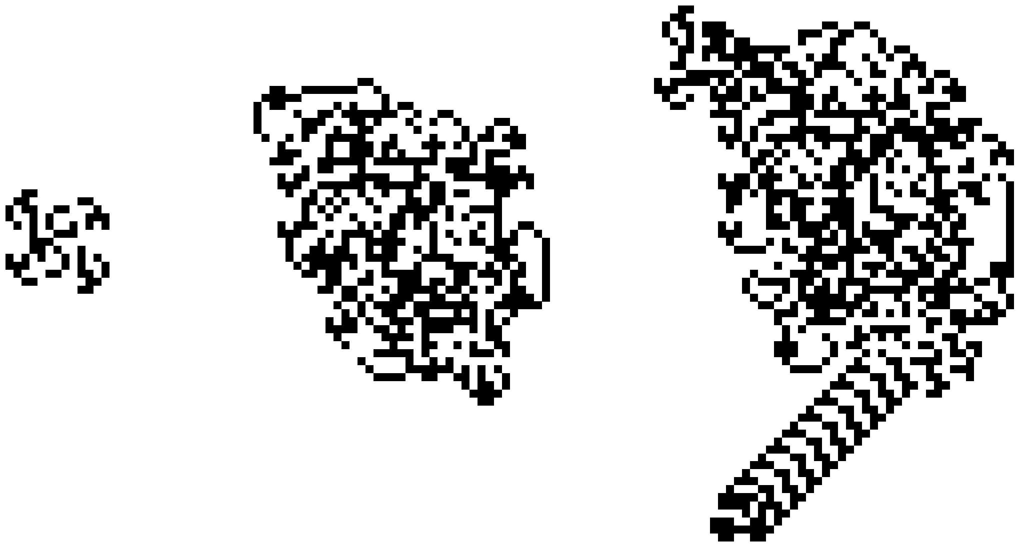
"Surprisingly complex behaviour for such a simple set of rules," I said.

"Yes. You see, Louise, Langton's rules are the 'Theory of Everything' for the universe that his ant inhabits. An anty-matter universe," he added apologetically.

"Yeah, and the rules predict exactly what it will do," Louise riposted.

"So an ant could reasonably wear 'Ant Rules' on a tee-shirt and proclaim the end of ant physics?" asked Dick.

"Not so fast," said Nathan. "Observe the strange sequence of shapes that the ant creates. For the first five hundred or so steps, it keeps returning to the central square, leaving behind it a series of rather symmetric patterns. Then, for the next ten thousand steps or so, the picture becomes very chaotic. Suddenly — almost as if the ant has finally made up its mind what to do — it builds what Jim Propp of MIT, who first made the discovery, calls a *highway*. It repeatedly follows a sequence of precisely 104 steps that moves it two cells southwest, and continues this indefinitely, forming a diagonal band." (See **Fig.2**)



Three stages in the infinite journey of Langton's Ant: symmetric, chaotic, and highway-building.

"Amazing," I said. Louise's face said *so what?*

"The really amazing thing," said Nathan, is that it *always* seems to end up building a highway, even if you scatter black squares around the grid before it starts."

"I think," said Granville, "that I can start it off next to an infinite line of black squares and it will just follow them off to infinity. If I get the spacing just right"

"Sorry. Any *finite* arrangement of black squares. But nobody has ever been able to prove that the ant always builds a highway."

"Can anything general be proved about what Langton's Ant does when it starts with any finite arrangement of black squares?"

"Yes. X.P.Kong and E.G.D.Cohen proved that the ant's trajectory is necessarily unbounded. It escapes from any finite region." See **BOX 1**.

"That's all very well," said Louise. "But what does it have to do with The Equation?"

"We *know* the Theory of Everything for Langton's Ant," said Nathan. "The rules. We set them up. But despite that, nobody can answer one simple question: starting from an arbitrary 'environment' of finitely many black cells, does the ant always build a highway?"

"So here the Theory of Everything lacks explanatory power?" I said.

"Precisely. It predicts everything but explains nothing. In contrast, the Cohen-Kong theorem *explains* why trajectories are unbounded."

"I can see several flaws in your argument," said Louise. "First, the Cohen-Kong Theorem is a *consequence* of the Theory of Everything, which therefore has at least some explanatory power. Next, your argument is founded on ignorance. Maybe tomorrow somebody will come up with a proof that ants always build highways."

"I agree. But it is making the Cohen-Kong consequence *explicit* that explains the unbounded trajectories. You can appeal to the uniqueness of the consequences of the Theory of Everything until you're blue in the face, but that alone won't tell you whether a bounded trajectory exists. Similarly, even though we know the Theory of Everything, we will have no idea whether highway-building is the universal pattern until somebody makes it an explicit consequence. Or disproves it."

"We constructivists have always maintained that nothing exists unless you can construct it explicitly," said Dick.

"Ah, but it all depends what you mean by 'proof'," said Inez. "Now in fuzzy logic —"

"Seems to me," interrupted Juliet, "that you're asserting an awful lot based on just one exceptional example."

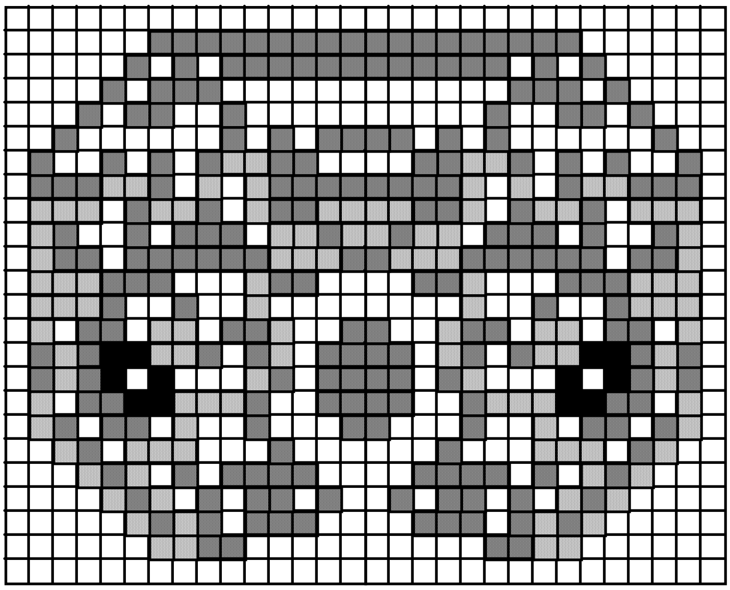
"Not really," said Nathan. "Langton's ant is entirely typical of rule-based systems. There are lots of generalizations, and they exhibit surprising behaviours — and even more surprising common patterns. You can have a lot of fun putting one or more ants into a chosen environment and seeing what they do. You can change the rules, and set up different environments — a hexagonal lattice, for instance, instead of a square grid. It's best done on a computer, where simple programs can implement the rules. I should add there's also a practical side to these ideas: they relate to questions in statistical mechanics about arrays of particles — 'ants' — that can exist in one or other of several states —'colored squares'."

"You mean particles and anty-particles?"

"Thank you, Tom. Now recently Greg Turk, and independently L.A.Bunimovich and S.E.Troubetzkoy, investigated generalized ants defined by a *rule-string*. Suppose that instead of just black and white the squares have n colors, labelled 0, 1, 2, ..., n-1. The rule-string is a sequence of n symbols **0** or **1**. When the ant leaves a cell of color k it changes it to color k+1 (wrapping n = (n-1) +1 round to 0). It turns right if the kth symbol is **1** and left if it is **0**. It moves one square on and repeats.

"Langton's original rules are summed up in the rule-string **10**. Some rule-strings give trivial ant-dynamics — for example an ant with rule-string **1** (or even **111**...**1**) travels forever round a 2x2 square. But any rule-string that contains both a **0** and a **1** must lead to unbounded trajectories, by the Cohen-Kong idea.

"Suppose for simplicity you start with a 'clean' grid — all cells in color 0. Ant **100** creates patterns that start out looking rather like those of Langton's ant — at first symmetric, then chaotic. After 150 million steps, however, it is still behaving chaotically. Does it ever build a highway? Nobody knows. Ant **110** does build a highway, and it takes only 150 steps to do so. Moreover, it needs a cycle of only 18 steps to create the highway, instead of the 104 used by Langton's Ant. Ant **1000** is relentlessly chaotic. Ant **1101** begins chaotically, but goes into highway-construction after 250,000 steps, using a cycle of length 388. Ant **1100** keeps building ever-more-complex patterns that, infinitely often, are bilaterally symmetric. (See **Fig.3**.)



A symmetric pattern produced at step 16,464 of ant **1100**.

So it *can't* build any kind of highway in the usual sense.

"I defy anyone to give a brief, simple classification of the behaviours of all of these generalized ants, or to predict from their rule-string just what their long-term behaviour will be — even if they all start on a clean grid."

Louise looked unhappy. "Yeah, but you haven't *proved* nobody can do that, Nathan."

"That's true," I said. "But only slightly more complex transition rules lead to examples such as John Horton Conway's game of Life. Conway proved that in Life there are configurations that form universal Turing machines — programmable computers. Alan Turing proved that the long-term behaviour of a Turing machine is undecidable — for example, it is impossible to work out in advance whether or not the program will terminate. Translated into Life terms, that implies that the question 'does this configuration grow unboundedly?' is formally undecidable. So here's a case where we *know* the Theory of Everything, *and* we know a simple question that it is provably impossible to answer on the basis of that Theory."

"Exactly," said Nathan. "So why do you think a real Theory of Everything, for our universe, can in any meaningful sense be an Ultimate Answer?"

"I dunno," said Louise, her faith temporarily shaken. She shook her head, then brightened. "It is a bit of an anty-climax."

BOX 1 The Cohen-Kong Theorem: ant trajectories are unbounded

It is easy to check that the Theory of Everything for Langton's Ant is time-reversible. That is, the current pattern and heading determines the *past* uniquely as well as the future. Any bounded trajectory must eventually repeat the same pattern, position, and heading; and by reversibility such a trajectory must be periodic, repeating the same motions indefinitely. Thus every cell that is visited must be visited infinitely often.

The ant's motion is alternately horizontal and vertical, because its direction changes by ±90_ at each step. Call a cell an H-cell if it is entered horizontally, and a V-cell if it is entered vertically. The H- and V-cells tile the grid like the black and white squares of a checkerboard.

Select a square M that is visited by the ant, and is as far up and to the right as possible, in the sense that the cells immediately above and to the right of it have never been visited. Suppose this is an H-cell. Then M must have been entered from the left and exited downward, and hence must have been white. But M now turns black, so that on the next visit the ant exits upwards, thereby visiting a square that has never been visited. A similar problem arises if M is a V-cell. This contradiction proves that no bounded trajectory exists.

FURTHER READING

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